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Distortion-Induced Vibration in Fan and Compressor Blading

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In contrast with the widespread tendency to rate the acceptability of inlet distortion patterns for airbreathing engine systems in terms of indices only stall related, this paper seeks to underline the importance of rating distortion characteristics specifically in their potential for inducing blade fatigue. In survey perspective, 8 mechanisms of distortion-induced vibration are described, only one of which has previously been discussed in the literature. While precision in each mechanism still represents a challenge to further research in steady and nonsteady aerodynamics, it is proposed that distortion characteristics can be identified specifically relevant to assessing blade resonance, random vibration in separated flow, and flutter, including a tentative approach to response in nonsteady distortion. A distortion index for blade vibration is defined both as an initial vibration design alert and for automatically assuring integration of design for inlet aerodynamics with that for blading.

I. Introduction

ONE wonders if highly important advances made in recent years by government and industrial research in the understanding and control of distortion-induced compressor stall margin degradation, may not have induced a preoccupation with this area with the result of slighting of another end product of a viable engines—life, as implied in freedom from distortion-induced blade vibratory fatigue. Underlying technologies of stall and blade vibration are indeed interdependent; even early formulations of nonsteady aerodynamics, initially developed for wing flutter, have contributed to stall understanding.[1-3] Blade aspect ratio trends benefitting stall margin usually help minimize vibration design challenges. Blade vibration signatures have contributed to performance gains and stall diagnosis. But extra severity distortion investigations tailored to stall margin demonstrations can also create blade vibration problems unrelated to actual service.

Preoccupied with performance aspects of distortion, are we not tempted to define designs and their applications relative to distortions acceptable in terms of indices that are only stall related? Usually performance degradation is given as a function of increasing distortion index in a low-to-moderate range. Structural damage is precluded up to

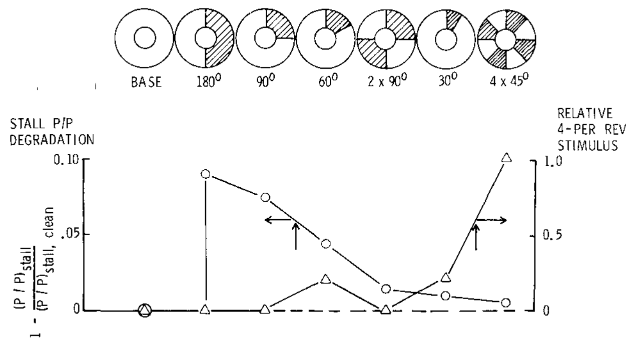

Fig. 1 Circumferential distortion implications for stall and resonant blade vibration are divergent.

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a much higher level. Yet stall-oriented distortion indices bear no necessary relationship to distortion-induced blade vibration. The physics of the two problems may actually yield trends directly in conflict. Figure 1 shows a progression of distortion patterns benefitting stall margin[3] but, as shown in the overlay, by elementary dynamics analysis, generates greatest potential for resonant blade vibration where stall margin is helped most.

A tally of General Electric experience with some 20 different engines built the past 25 years indicates only 2 hardware changes required because of distortion-induced blade vibration: the shrouding of a fan blade because of vibration encountered in crosswind inlet distortion; and a blade frequency change in a multi-stage compressor because of a type of internal distortion. Long-term design sensitivity to the implications of distortion for blade vibration, one may deduce, has contributed to a so far encouraging circumventing of such problems.

But operating environments are becoming more hostile and complex as seen in high bypass fans, vertical take-off engines, and high Mach applications. A fan test time-averaged crosswind distortion pattern is shown in Fig. 2. A combined time-averaged and dynamic distortion pattern is illustrated in Fig. 3. In parallel, economically successful vibration control for long life engines hinges on obtaining increasingly accurate definitions of increments in vibration. Fortunately technology is advancing in this area, e.g. for the direct calculation of resonant vibration due to circumferential distortion. Pending further advances, however, we must assure an "automatic" application of existing technology and generalized experience to avoid "inconceivable" failure by oversight implicit in distortion index limits related to stall alone.

Accordingly, the text of this paper contrasts a typical stall-related index of distortion with aspects of distortion one must consider mechanism-by-mechanism for blade vibration. Beyond the one mechanism previously recognized as sufficiently significant for treatment in the literature, resonance of rotor blades, 7 others found significant in modern design are discussed: 1) resonance; 2) random vibration generated by excitations associated with cascade separation; 3) its combination with resonance maximized at a critical operating temperature, resonance *indirectly* generated by distortion accentuation of these mismatched stator cascades; 4) blade instability, or flutter, induced by distortion in stages stable without distortion; 5) inlet gust excitation; 6) upstream turbulence; and, 7) oscillatory inlet flows.

Finally, an index of distortion related to blade vibration is given as a design alert, and an inter-disciplinary communications device for minimizing oversight in the application of guides is already available. Discussion is limited to blade vibration in the prestall operating regime of fans and compressors; for stall, referred to fans and compressors as a whole rather than local "cascade stall," is a generally unacceptable operating condition. Blade response in fan or compressor stall is a specialized subject in itself.

II. Distortion Indices for Stall vs Some General Considerations for Blade Excitation Effectiveness

Efforts to predict stall degradation as function of inlet distortion have evolved a number of indices for describing distortion patterns ranging from the simple overall parameter $(P_{T\text{MAX}}-P_{T\text{MIN}})i/P_{T\text{AVG}}$ through indices weighting the spatial extent of pressure decrements in various ways, to present day approaches of increased complexity. In general, however, a given arbitrary pattern is considered as the combination of superimposed radial and circumferential components.

In one treatment, the inlet face is envisioned as 5 equal-area, concentric annuli for which component annular ring indices are computed; $(IDC) = (P_{T\text{AVG}} - P_{T\text{MIN}})i/P_{T,\text{FA}}$ and $(IDR) = (P_{T,\text{FA}} - P_{T\text{AVG}})i/P_{T,\text{FA}}$ (FA = face average) (i = ring location) with overall percent stall degradation expressed in terms of weighted values of these 2 component indices. Weighting factors for circumferential components of distortion reflect, for example, the relative area extent of pressure decrements and the number of circumferential lobes, the two inner and outer diameter ring indices generally being given priority even to the exclusion of these at the mid-annulus region. Weighting factors depend on such compressor parameters as the slope of the load-flow characteristic at constant speed, the relative cascade loading at hub vs tip, etc. Time-unsteady distortion is generally reduced to equivalent steady distortion for stall correlation for "instantaneous" pattern resident time of the order of one revolution.

By contrast with variables affecting stall index weighting factors, distortion-induced blade vibration severity depends upon the effectiveness of energy input to blades—in essence a coupling of the distribution of excitation forces and blade mode shape. For a given excitation intensity its effectiveness in generating response will be proportional to the applicable Lagrange generalized force; e.g., for a blade without twist, $\int (u/u_o)(w(z)/w_o)dz$, where u/u_o = normalized modal displacement; $w(z)/w_o$ = normalized distribution of excitation force per unit span, unit force/unit span = spanwise dimension, and = blade length.

Excitation forces ultimately depend on practically the same fundamental research topics of nonsteady aerodynamics as do considerations of stall initiation, for their evaluation except that blade excitation forces relate to the characteristics of the local cascade while multistage compressor stall initiation depends upon conditions of flow dynamic equilibrium and stability among cascade systems. In the immediate context of blade vibration-related definitions of distortion, however, the simple concept of effectiveness of energy input provides a powerful tool for pattern discrimination even in this era when definition of motion-dependent nonsteady aerodynamic forces and moments is still being progressively developed according to the familiar sequence: isolated airfoil, incompressible; cascade, incompressible; cascade flat plate, subsonic; cascade with finite thickness and camber, subsonic; cascade flat plate, supersonic, etc.[12-14,16,17]

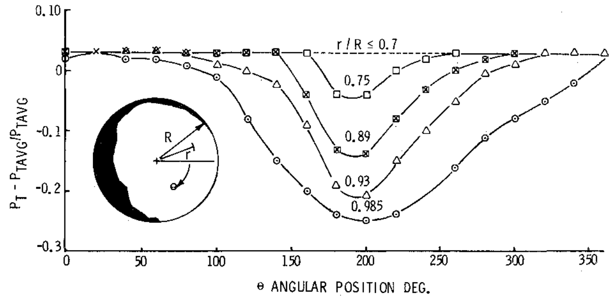

Fig. 2 A severe crosswind distortion fan inlet, high fan speed, plane standstill.

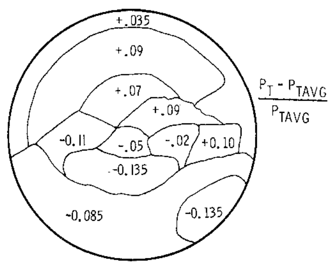

Fig. 3 Arbitrary test distortion including anticipated dynamic component.

Pending availability of such nonsteady aerodynamic forces in detail, highly important conclusions for the amplication of both analytical and empirical response characteristics of both forced and self excited vibration have been made by weighting radial distributions of distortion in terms of a blade's "critical span" between stations z_a and z_b where net energy input to the blade is practically that for the entire blade, in essence where

$$\int_{z_a}^{z_b} \frac{u}{u_0}\frac{w(z)}{w_0}\,dz \approx \int_0^1 \left(\frac{u}{u_0}\right)\frac{w(z)}{w_0}\,dz$$

The approach is similar to the practice of a much earlier period in categorizing calculable wing flutter characteristics in terms of parameters obtaining at an equivalent reference station.

Overall characteristics of inlet distortion relate to blade vibration in general as follows: 1) Circumferential components of distortion generate flowfields capable of inducing resonant response to rotor blades, both instability (flutter) and severe random excitation, in stator vanes; 2) Radial distortion is of particular interest for its potential for generating flutter primarily in rotor blades but also in stator vanes. Blade response vs distortion pattern is discussed mechanism-by-mechanism in what follows.

III. Rotor Blade Resonance Induced by Circumferential Distortion

From the earliest days of airbreathing engine design it was universally recognized that circumferential distortion patterns are capable of generating severe resonant response in rotor blades. Design evolution has not diminished the importance of this mechanism. High bypass fans operating in crosswind are capable of experiencing both familiar lower-order blade mode response and blade-disc system mode response of high intensity as well.

And high-intensity sectors of inlet distortion, too narrow in angular extent to be of concern for stall degradation, are capable of generating serious higher mode response in multistage compressor blading. Considerable progress has been made in analytical prediction of quantitative blade vibratory stress given only inlet pressure distribution, blade geometry, and rotor speed, with key ingredients being a compressible flow formulation of nonsteady aerodynamics for cascades and an advanced treatment of blade and disk dynamics. Illustrative examples of resonant vibration and its relationship with the distribution and intensity of inlet distortion may be of interest.

A. Excitation Harmonics and the Campbell Diagram

The reasoning is familiar. A circumferential distribution of pressure at a given radius can be represented as a Fourier series of terms in which variables are the angular position and the harmonic. At a given radius, for example, a single sector centered about the origin $\theta = 0$, of angular width $2\Delta\theta$, and of intensity $\bar{P}$ can be represented, one recalls, by the series

$$p(\theta) = \frac{\bar{P}}{2} + \frac{2\bar{P}}{\pi}\sum_{n=1}^{\infty} \frac{\sin n\Delta\theta}{n}\cos n\theta$$

Now to the extent that the resulting downstream flowfield results in perturbations of velocity relative to the leading edge of a rotor blade of the same form (actually, a crucial consideration and increasingly so for downstream stages and fans preceded by inlet guide vanes), linear nonsteady aerodynamics indicates complex oscillatory forces, $\bar{f}$ acting on the blade cross section in the form

$$\bar{f}(t) = K\bar{P}\sum_{n=1}^{\infty} \frac{\sin n\theta}{n}\cos n\Omega t$$

as the blade sweeps out angular location ν as the product of constant angular speed Ω and time t; and where K is the product of P, air density, blade chord, and a function of excitation reduced frequency (chord x$n\Omega$ ÷ mean relative velocity).

Elementary analysis indicates that harmonic excitation $f(t)$ will generate significant vibratory stress σ_v only for the resonant conditions at which excitation frequency coincides with a blade normal mode frequency ω_N. Total resonant response will be a linear sum of resonant natural mode responses, each proportional to the relevant Fourier coefficient—$P(\sin n\Delta\nu/n)$ the mode-dependent effectiveness coefficient

$$\left[\int_0^1 \frac{P}{P_0}\left(\frac{u}{u_0}\right)d\eta\right]_N$$

and inversely proportional to total aerodynamic and mechanical damping in that mode, δ_N. Thus

$$\sigma_v = X\bar{P}\sum_{n=1}^{\infty} \frac{1}{\delta_N}\left(\frac{\sin n\Delta\theta}{n}\right) \times\left(\int_0^1\left(\frac{P}{P_0}\right)\left(\frac{u}{u_0}\right)d\eta\right)_N \varphi_N \cos(n\Omega t - \varphi_N) \quad (1)$$

where, of course, X is a geometry dependent constant; P/P_o and u/u_o are, respectively, spanwise distributions in sector pressure and the blade vibratory displacement distribution normalized for normal mode, N, over fractional span, η. The variable α_N is the mode-and-damping-dependent phase angle; and ϕ_N is a mode-and-speed-dependent measure of centrifugal field restoring action. For simplicity only an untapered blade without twist is illustrated here. It is conventional to represent conditions of resonance, or $n\Omega = \omega_N$, by a superposition of blade normal mode frequencies vs rotor speed and of a fan of multiples of rotor speed in compatible units, a representation denoted a Campbell diagram in deference to the pioneering paper of Wilfred Campbell.[21] An illustrative Campbell diagram is given in Fig. 4 for a research compressor rotor blade indicating resonance speeds for the first four blade modes. However Campbell diagrams in themselves say nothing about the intensity of response.

Analysis outlined in Eq. (1) implies blade response directly proportional to the intensity $\Delta P/P$ of a given circumferential pattern, and to its IDC counterpart. That resonant response tends to be proportional to $\Delta P/P$† is not a new observation. References 4 and 5 have discussed this point, and as early as the classic paper of Pearson,[6] the evaluation of blade oscillatory forces was outlined in terms of perturbations of velocity relative to a rotor blade

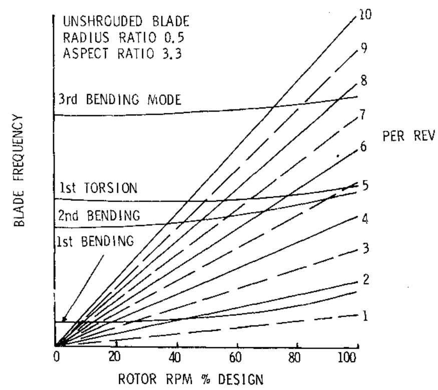

Fig. 4 Illustrative Campbell diagram blade frequencies relative to multiples of rotor speed.

†More exactly, velocity perturbations $\Delta V/V$.

section, deriving velocity and incidence perturbations as proportional to $\Delta P/P$ for sector distortion, assuming circumferentially constant static pressure, neglecting pattern redistribution with axial distance. Early papers treated resulting blade forces in terms of lift coefficients quasi-static or those nonsteady aerodynamic coefficients obtained by Theodorsen[9] for the case of an isolated airfoil in incompressible flow.

Blade fatigue prevention, our ultimate interest in distortion-induced vibration, depends upon absolute magnitudes. Two reference points are of particular interest: 1) blade first bending mode response at 2 per rev to low-intensity, steady 2 per rev patterns; and 2) blade higher mode response to high-intensity sector distortion of small angular extent. Figure 5 presents data of the first type, for a range of $\Delta P/P$. Campbell diagram characteristics in Fig. 4 indicate this resonance occurred at 50% design speed.

To convey response intensity in terms directly relevant to steady-state design parameters, vibratory response is expressed in terms of bending stress for air loads at resonant speed, with stress corrected for centrifugal field stiffening action. Response magnitudes are high as expected. Data show significant response even for nominally "clean" inlet operation. Trends in blade response as a function circumferential extent of distortion at constant $\Delta P/P$ suggested a flow redistribution as the cause of deviations from results directly attributable to a Fourier analysis of pressure distortion $(\Delta P(\theta)/P)$, despite the distortion's being measured just ahead of the rotor blade by circumferential traverse. Indeed, the gross index $(P_{T\mathrm{MAX}}-P_{T\mathrm{MIN}})/P_{T\mathrm{MIN}}$ appears the most significant parameter.

These data are obtained on our 1960 Aeromechanical Compressor Program, single-stage research compressor using screens of progressively higher distortion intensity and of 2 sector pattern. More recent data from fan crosswind tests parallel these results in point of linearity of average response vs $\Delta P/P$ in one fan, at a part-speed, 2 per rev, first bending resonance; in another fan, at part-speed, 3 per rev resonance in the second bending mode. Tests proved these fans free from fatigue concerns as a result of having designed to limit low order per rev resonance to speed ranges of inherently low velocity head and corresponding low distortion magnitude.

Data for high-intensity narrow-sector distortion, given in Fig. 6, are noteworthy, by contrast, in illustrating higher mode response at a succession of per revs in accordance with implications of simple Fourier analysis. Related data indicate such stimuli are capable of generating blade fatigue.

Observations parallel those obtained from multistage compressor blades responding to an unusual, but real, distortion pattern made up of 2 full span distortion sectors of only 5° arc width and separated by an arc of 42.5°. As in other cases we have observed, intense, narrow-sector distortion tends to persist through several stages: in this instance, with a sharpness of definition permitting simple

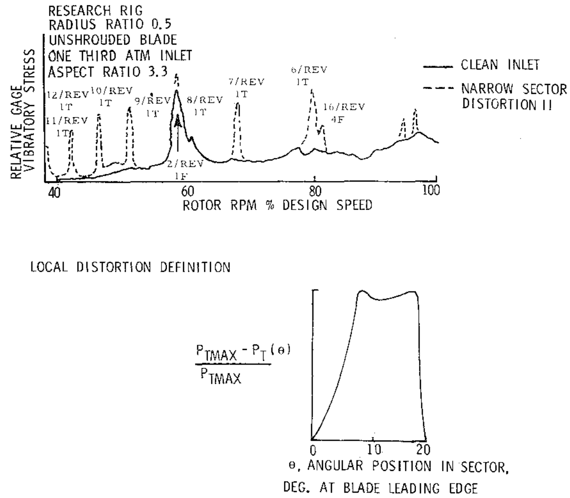

Fig. 6 Intense circumferentially localized distortion sectors excite multiple blade resonances.

harmonic analysis to represent closely the reality of relative magnitudes of blade response severity over a succession of resonant peaks. An overlay of test data and analysis is given in Fig. 7. Absolute levels of blade response approached fatigue endurance.

Characteristics cited in Figs. 6 and 7 introduce a specific caution relative to views that distortion involving higher harmonics, and associated higher resonant modes are of no significance. These examples also illustrate both the need for distortion pattern definition beyond that normally provided by stall-related indices; and that conclusions of practical significance for blade vibration can be obtained when such definitions exist.

There are related characteristics of distortion significant for vibration but of lesser importance for stall. Consider the significance of multiple sectors, inlet swirl, and the radial distribution of distortion intensity within a given sector.

B. Sector Width and Multiple Sectors

In Eq. (1) the factor $(\sin n\Delta\theta)/n$ implies the frequently reiterated observation that the strength of single sector excitation varies inversely with the harmonic, or multiple of engine rotor speed n. Worst combinations of angular extent and strength corresponding for a given harmonic follow the sequence given in Table 1.

But if for a given harmonic n there are an n identical equispaced sectors (e.g. 4 45° sectors), their total resultant excitation strength is as great as that produced by 2 90° sectors, a pattern known to be capable of breaking blades

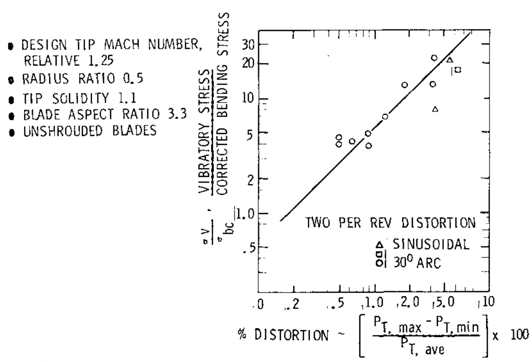

Fig. 5 Gross characteristics of blade resonance at 2/rev with 2/rev sector distortion.

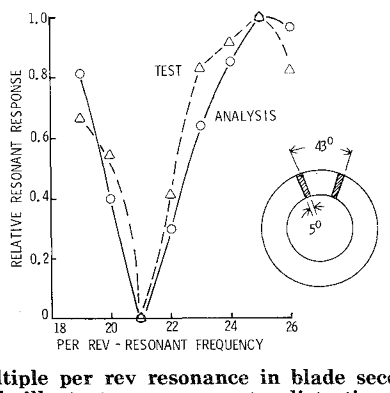

Fig. 7 Multiple per rev resonance in blade second bending mode illustrates narrow sector distortion harmonic interaction by addition and phase cancellation.

Table 1 Worst combinations of angular extent and strength corresponding for a given harmonic

n	1	2	4	6	10
$2\Delta\theta$ deg	180°	90°	45°	30°	18°
Excitation relative strength	1.0	0.5	0.25	0.166	0.10

at fairly low levels of $\Delta P/P$ (cf. Fig. 5). Again, this is higher modes.

C. Inlet Swirl

Inlet swirl, and its nonradial skewing of sector boundaries apparently does not affect stall margin degradation. But such skewing needs to be considered for blade vibration. An analysis of resonant response in the presence of nonradial inlet strut wakes and of stator wakes in free vortex turbines[7] has been adapted to show the influence of sector skewing on resonance severity of idealized rotor blade torsional modes. Figure 8 indicates that a skewing of $\lambda = 30°$ would double, as opposed to the nonskewed case, second torsional mode response resonant with the fifth harmonic of the distortion sector.

D. Spanwise Distribution of Distortion

Again, Eq. (1) contains an integral which measures the effectiveness of coupling of resonant mode shape with spanwise distribution of distortion. Figure 9 illustrates this consideration in terms of the first two torsional modes of idealized uniform blades, one a cantilever, the other, a tip-shrouded configuration. Relative blade response levels in this figure are independent of distortion sector angular extent, the resonant per rev and whether the pattern is a single- or a multi-sector configuration. Radial distribution alone is considered. Response levels are presented relative to a radially uniform sector as an excitation base. As expected, circumferential distortion concentrated at the hub is relatively not severe. Slightly surprising, however, is the result that a sector stimulus, concentrated over one-third annulus height and centered at the pitch region, is as effective a stimulus for the first torsional mode of a tip-shrouded blade as is a sector concentrated over the outer one-third annulus height for the first torsional mode of a cantilever blade. Rarely are compressor stall tolerance demonstrations based on limiting case distortion patterns concerned with this latter profile.

E. Real World Resonance Analysis

The foregoing involve research compressor characteristics and deductions from analysis of idealized blading. But what of quantitative success in the analysis of high bypass fans for example, operating in a crosswind environment? A calculation of such blade stress response levels, carried out in 1968 on the basis of first principles theory alone for the arbitrary distortion and blade geometry, was in encouraging agreement with observation.

The transonic fan involved a titanium cantilever airfoil, of aspect ratio 4.4 and hub-to-tip radius ratio of 0.36. Crosswind vibration was everywhere flutter-free but, because of specific crosswind distortion patterns not foreseen in design, required the addition of a midspan shroud, specifically to suppress response levels excessive for unrestricted service. Vibration response generally involved components of multiple-mode random response and discrete coupled blade-and-disc system modes, the worst being the second 5-diameter mode.

Resonant stress response in the first 3-diameter blade-and-disk system mode, filtered to remove response in other modes of overall activity, was chosen for comparison with analytical calculation. The data are for the configuration after shroud addition, the location and stress reduction magnitudes having been projected by less elaborate calculations. At the fatigue critical point, just above the shroud fillet, measured stresses ranged from ±16,200–±17,200 psi in crosswinds from 20–40 knots in the range of 74.7%–79% design speed. Computed resonant stress response was ±16,000 psi.

Some specifics of methodology may be of interest relative to earlier published work: 1) Measured distortion levels and distributions were transformed into a Fourier series definition, over the full span of the blade leading edge, of axial and tangential relative air velocity perturbations cast in complex variable form. Spanwise distribution of phase (sector skewing) and intensity were automatically reflected in the formulation.

2) Nonsteady aerodynamics forces corresponding to those distortion-induced velocity perturbations were directly computed from the compressible flow formulation of cascades constructed by Dr. R. Mani[8] of the General Electric Corporate Research and Development Center as part of our 1968 aeromechanics development program. This particular formulation, it will be seen, bypasses limitations implicit in earlier published work in which nonsteady aerodynamic forces were based on quasi-static load coefficients or derived from Theodorsen's[9] theory of oscillating, isolated airfoils in incompressible flow.

3) Stress and dynamics for the blade-and-disk system were obtained by a proprietary, computerized analysis, cast in complex variable form precisely to facilitate solu-

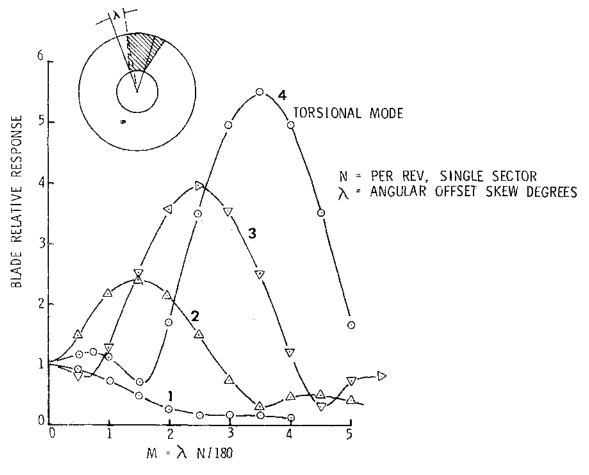

Fig. 8 Distortion induced blade resonance relative response vs sector skewing cantilever blade–torsional modes.

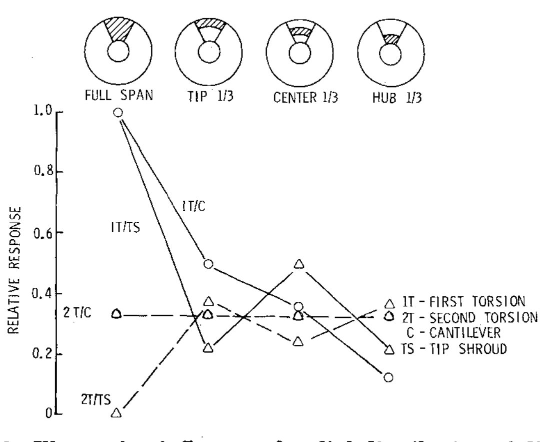

Fig. 9 Illustrative influence of radial distribution of distortion on torsional blade mode resonance.

tion of linear aeroelastic problems, and solved by direct integration of relevant differential equations, bypassing approaches dependent on an energy integration over the blade span as in Refs. 5 and 10.

This same dynamics formulation, found highly versatile since its first application in aeroelasticity 2 decades ago was also used in the study of blade response to the massive inlet gust loads discussed in Sec. VIIA. Accuracy of natural frequency prediction may be inferred from Ref. 11.

IV. Distortion Induced Blade Instability ("Flutter")

Cautiously accelerating a fan engine in an extra-severity performance test one day almost a decade ago, the throttleman initiated emergency "drop-speed" at 56.5% design rpm. (He was especially alert, briefed in advance to expect just such a command at "approximately 55% speed.") In context, fan thermodynamic performance had been sought in the presence of tip radial distortion of extraordinary severity. Speed backoff coincided with fan vibrations suddenly climbing precipitously in blade instability generated by that distortion. This factual incident is recounted to indicate that frontier technology subject though this topic remains yet for refinement, blade stall flutter was then sufficiently defined to forecast correctly specific distortion-induced blade instability in a fan already proven stable, even in the most severe crosswind distortion. Hence, it is suggested in this section that radial distortion can generate blade instability; and that patterns of sufficient intensity can be distinguished from those which do not.

Blade instability, or "flutter," prevention is a continuing design challenge even for clean inlet conditions. At least 5 specific types can be distinguished. Experience in various designs and operational regimes has shown instability to be possible in each of the first 3 modes of cantilever blading; the first 2, for shrouded blades. Our interest here is not a treatment of blade instability as such, but whether and under what conditions distortion may provoke instability in blades stable with clean inlet flow. To avoid confusion with some literature of a much earlier time period, "instability" denotes a truely self-excited vibration with aerodynamics/blade elasticity feedback, and air stream forces as a function of blade motion, as opposed to random excitation type vibration generated by cascade separation spectra.

In our experience, at least, blade instability has been avoided in distortion as it naturally occurs in actual installation, abusive flight test, and service. For one engine, special flight tests were made precisely to guard against distortion of unique character. Concern involved need for precise definition of the distortion and, particularly its dynamic character, rather than blade stability characteristics as such, or their interaction with distortion as coupled in criteria given here.

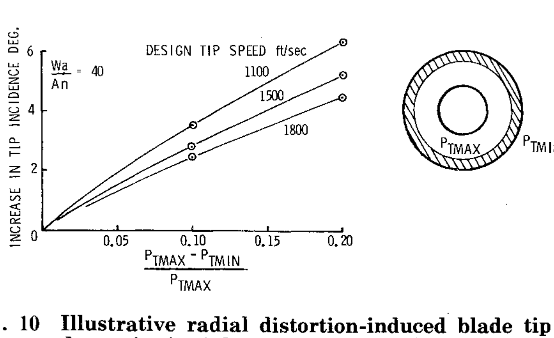

Fig. 10 Illustrative radial distortion-induced blade tip incidence rise (no inlet guide vanes).

Criteria: To distinguish distortions having instability-generating potential from those that do not, two considerations we find to be critical are: 1) Rotor blade stability boundaries valid for clean inlet flow apply in the presence of distortion when stability parameters dependent on incidence, Mach number, and relative air velocity are taken to be the circumferential average, radial station by radial station. Hence, circumferential sector type distortion affects rotor blade instability boundaries only as it indirectly changes circumferential average (time-average) flow.

2) To constitute on adequate trigger, distortion must create time-average (circumferential average) flow conditions consistent with clean inlet instability boundaries over the "critical span" for a given mode: that portion of the span over which mode displacement is high, e.g. the outer 25% of an unshrouded blade for first blade bending, first coupled system bending mode, or first torsional mode; 50–75% span for the first torsional mode of a tip shrouded blade. Figure 10 gives a first approximation calculation of incidence excursion, for a fan without guide vanes, as a function of radial $\Delta P/P$.

Indicated incidence changes are comparatively large in a stability context. Stability boundaries are empirically determined. But the form of the integration and its physical meaning permit practical definition of critical span zones and related specific distortion pattern assessment. Thus the concept of a "critical span" for instability generation directly parallels considerations of effectiveness of energy input discussed previously relative to resonance. For, except in certain cases of supersonic flutter[12-14] and blade system instability in the subsonic regime, nonsteady aerodynamic coefficients are not yet calculable analytically to permit an explicit integration over the blade span as in blade resonant response with subsonic cascade flow.

Criteria 1 and 2 derive originally from a series of single stage transonic compressor tests specifically directed to the question of blade instability in the presence of distortion, a part of our 1960 Aeromechanical Compressor Program of applied research in blade vibration. Illustrative data are shown in essential form in Fig. 11. Experience from engines support these criteria.

With radial distortion, flow redistribution must be examined for implications of increased velocity in "clean" zones. Zones of increased velocity must be examined relative to penetrating the boundary for supersonic low-incidence instability in a fan at high speed. Similarly in a less probable and more difficult to assess situation, flow redistribution must be checked for penetration of the negative incidence stability boundary for intermediate stages of multistage compressors.

While large sector distortion has the potential for inducing stator vane instability in the manner of radial distortion for rotor blades, we have never observed it on test

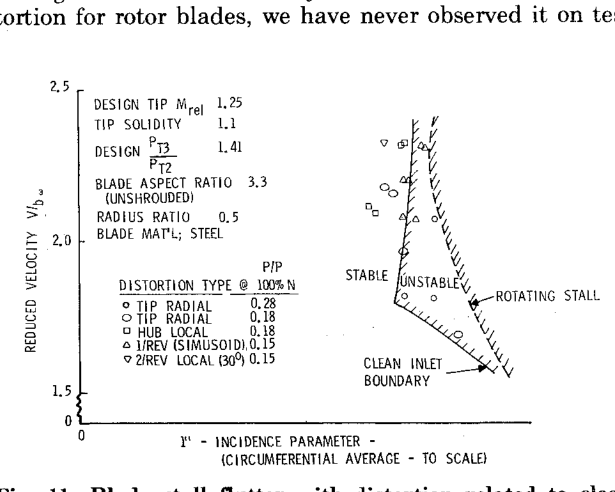

Fig. 11 Blade stall flutter with distortion related to clean inlet stability boundary.

nor experienced vane fatigue failures attributable to it. Hub radial distortion, or large sector distortion concentrated at the hub, attenuating slow enough to produce large incidence excursions in unshrouded vanes of multistage compressors require design assessment for distortion-induced instability. Fortunately fan stator vanes and the first few stages of multistage compressors are shrouded for other reasons; e.g., bird strike excursions.

A. Unsteady and Transient Distortion-Induced Blade Instability

The foregoing implies time-averaged distortion. It relates directly to time-unsteady distortion only as experienced by rotor blades traversing circumferential sector distortion. Indirectly time-averaged circumferential distortion carries over implications for time unsteady distortion. Indeed one unusual flight test documented discrete point "instantaneous" levels of $(P_{MAX}-P_{MIN})/P_{MAX}$ of at least 0.5 with no suggestion of blade instability. Considerations of "critical span" and related blade energy input together with experience with circumferential distortion, leads to the expectation, so far experience-confirmed, that time-unsteady distortion has potential for instability generation *only* when it is of a very special type, the character of which satisfies the following two conditions simultaneously.

1) The "instantaneous" pattern must be at least as severe as one required to cause blade instability if applied in time-steady distortion. And as previously indicated (e.g., radial distortion, coinciding with a "critical span") it must be sufficiently intense to cause incidence/velocity shifts to the stability boundary, or so extreme a circumferential distortion that circumferential average flows are equivalently affected.

2) The entire pattern fulfilling condition 1 must persist for a finite time, the "critical time." It is deduced to be somewhat greater than one rotor revolution. Tentatively one estimates that the critical time is equivalent to 5 first- or second-bending mode natural frequency cycles or 5 torsional mode cycles, the relevant mode being chosen according to the distortion pattern's radial distribution and clean inlet blade stability characteristics. In rotor revolutions, these latter critical times are approximately equivalent to 1–2 revolutions, depending upon the relevant mode. The tentative critical time range beyond a 1 revolution equivalent derives from considerations of the basic nature of an instability together with numerous observations of one type of transient blade instability.

Distortions in actual installation, as opposed to idealized test patterns, inherently incorporate both time average loss and fluctuating components. The foregoing suggests that distortion-associated fluctuating components consistent with criteria above are rare but identifiable. Those having the highest potential of satisfying these criteria involve more nearly quasi-periodic separations from massive surfaces ahead of the inlet as may be conceivable for certain lift engine configurations.

By contrast, idealized screen generated distortions tend to be more severe than those for actual installations even for the same nominal time-average intensities. Screens intended to demonstrate stall capability for a "frozen-time," combined steady-and-unsteady distortion pattern require careful examination lest they introduce blade instabilities unreal with respect to the actual "instantaneous" patterns being simulated. Additionally, uniform screen radial distortion may generate quasi-periodic vortex shedding wakes of intensity great enough to excite blades to high stress levels, thus creating an artificial vibration problem. One such event comes readily to mind, a case confused initially with blade instability. Stall-distortion sensitivity programs require tailoring to achieve balanced benefits for vibration as well as performance.

For transient pressure patterns, as opposed to those quasi-periodic or randomly fluctuating, steady flow criteria apply when the "critical time" is exceeded. An inlet pressure depression equivalent to a transient loss of inlet recovery, blanketing the whole annulus and of intermediate magnitude, is the most significant transient distortion; i.e., a depression insufficient to cause stall but sufficient to cause blade incidence migration to its instability boundary. In practice, inlet design valid for compressor stall protection (and considering "tolerance") should preclude this situation. Transients of supersonic inlets appear relevant.

Technology areas for advancement: 1) Parametric, precision experimental efforts to parallel related analytical developments are implicit in topics discussed in this paper, "Critical time" definition with refinement for blade instability initiation, buildup; 2) Repeated pulsed application of critical incidence/velocities as a function of pulse period.

V. Separated Flow Vibration

What have they in common? Severe vibration is observed in exit guide vanes of a development fan in crosswind environment; exit guide vanes of another fan is undergoing high intensity, screen sector distortion tests with vibration levels of vanes in the sector wake three times the level of vanes outside the sector wake; and on-the-spot calculations are proving reported levels of multistage compressor vane vibration were incorrect.

The mechanism of vibration in each case was that we designate "separated flow vibration," resulting from spectra of random driving forces of cascade separation. This was previously denoted "heavy loading" in Ref. 22, now simply expressed as "separated flow vibration" to include negative incidence, with the designation suggesting the remedy. Cascade flow separation can be generated by a distortion flow decrement with attendant blade or stator vane vibratory response. A circumferential average distortion affects rotor blades; circumferentially localized, stator vanes. In general, blade and vane vibratory response of this type is fixed by corrected speed, N/T_{inlet}, for a given inlet pressure.

It is proposed that severity of such vibration can be related to the intensity and distribution of specific distortion patterns. In contrast with a true instability, such vibration is forced, stable, and follows its own laws of response intensity quite distinct from those of a true resonance or instability; e.g., with respect to air density and reduced frequency. Such distinctions we recognized in the behavior of a single stage research compressor in 1952, formulating design implications in 1953, implications not previously discussed in the open literature except for the treatment of Whitehead.[15]

Some noteworthy characteristics are these: 1) Excitation frequency spectra are continuous, with amplitude dependent on cascade geometry generally decreasing in intensity with increasing reduced frequency. Attempts at response limitation by simple frequency detuning are ineffective; 2) Excitation amplitude is dependent on cascade aero loading, with a maximum at some critical positive incidence as much as ten fold that for low incidence or for incidence so high that full cascade stall occurs. A comparable amplitude-incidence characteristic obtains in the negative regime; 3) Response intensity is governed primarily by cascade geometry and operating condition and to secondary degree by turbulence of the inlet air stream; 4) Response to this mechanism is of primary importance for inlet stages in multistage compressors; 5) Response signatures evidence distinctive amplitude modulation with definite maximum repeatable in magnitude for a given operating condition, repeatable in time only in a statistical sense.

An isolated airfoil characteristic is shown in Fig. 12 together with its typical counterpart (Fig. 13) observed in Stage 2 of a multistage compressor, with vibratory magnitudes given for illustrative "feel" as a multiple of steady-state air load stress corrected for centrifugal field restorative action. Such vibration in front end multistage compressor stages is an inherent consequence of part-speed, aerodynamic stage-mismatch even without distortion. Rotor blade vibration sensitivity to distortion depends upon the portion of the response-incidence characteristic involved; e.g. if separation is already inherently severe, distortion-induced aggravation of separation may produce no further vibratory response. In essence, blade and vane "separated flow vibration" response to inlet distortion depends on the application of the cascade's inherent vibration/aero loading characteristic evaluated in terms of local environment (e.g., incidence over the critical span) as determined by the distortion.

In physical terms, gross blade "strength," as implied by time-average air load stress (aspect ratio, thickness to chord ratio) and shrouding, is the key parameter. Encouraging projections of peak vibratory response level, σ_v have been obtained from a simple relation in the form

$$\sigma_v = K(C, \bar{\omega})F(i, M)\sigma_a^{1/2}$$

where $K(C,\bar{\omega})$ is a function of cascade geometry C and reduced frequency $\bar{\omega}$; $F(i, M)$ is a function dependent on incidence i and Mach number M; and σ_a is a time average state, air load reference stress at incidence i and Mach number M.

V. Combined Separated Flow Vibration and Indirect Distortion Resonance

Simultaneous, multiple-mode blade response[11] is surely familiar to all fan and compressor designers; less so, perhaps, is simultaneous multiple-mechanism response in the same mode, or a resonance with response seemingly temperature-dependent, still less a *radial* distortion amplifying vane wake generated resonance in rotor blades.

Such a combination is that of blade resonance at a discrete physical rotor speed together with random excitation response due to blade cascade separation, not necessarily severe relative to performance and, for a given inlet pressure, varying with corrected speed, N/T_{inlet}. In terms of combined mode response, one finds the implications of a blade resonance temperature-dependent, with combined peak response sufficiently discrete that test demonstration requires pretest operating point definition for experimental verification. The example given here illustrates combined mode response together with distortion-magnified inlet strut wake intensity, response in the second bending mode, resonant with the second harmonic of the strut wake passing frequency.

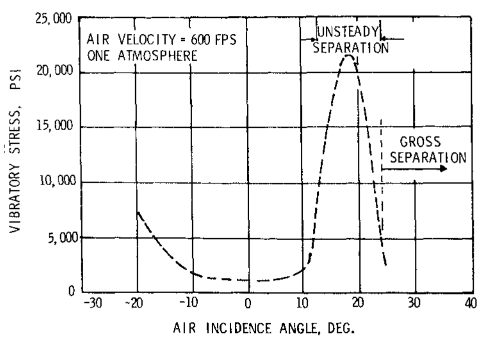

Fig. 12 Illustrative isolated airfoil separation flow vibratory response vs incidence.

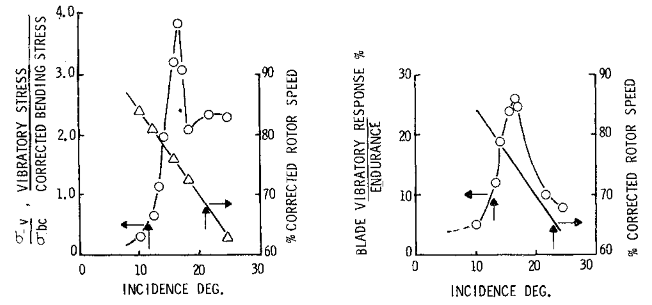

Fig. 13 Illustrative cascade separation—excited blade vibration—first bending mode stage 2 multistage core of fan engine.

A relevant case history involves tests run on the high pressure compressor component of a full fan engine, at standard inlet temperature, with and without radial distortion, screen-generated, at the hub. This was a limiting case demonstration of performance margin. Vibratory stress for the first stage blade, with magnitude expressed relative to fatigue endurance limit, is shown in Fig. 14.

Cascade separated flow vibration in the first bending mode peaked at 80% rotor speed. At 90% rotor speed, wakes from struts in the transition duct between fan and core compressor excited the second bending mode in resonance at the second harmonic of strut passing frequency. On test the 2 vibration peaks were discrete. But at some higher inlet temperature they would superpose; i.e., at the inlet temperature where the 80% corrected speed would be achieved at the physical speed corresponding to strut resonance. That core compressor inlet temperature was 196°F. Because of specific differences in modal stress distribution between first bending and second bending modes, only 54% of the second bending mode's peak was effective in contribution to the combined mode percent endurance level; the combined mode test condition was acceptable and more severe than for actual installation distortion. This case illustrates as well the case of distortion-magnified wake excitation.

VI. Special Mechanisms

A. Gust Excitation

A special form of blade excitation has been found in crosswind distortion of high bypass fans—massive aerodynamic impulse loads, quasi-periodic in the order of 1 impulse per 1500 blade system cycles, generating discrete-incident transients in multi-mode blade response. Its massive impulse character is noted in contrast to the concept of treating vane wake perturbations and attendant rotor blade resonance in the manner of wing gust analysis.

"Large scale" flow instability involving ground vortices and inlet lip transient separation appear to be involved.

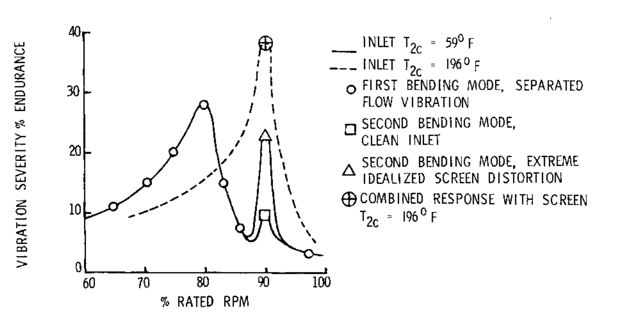

Fig. 14 Illustrative example: inlet temperature implications of combined mode vibration with distortion.

Gross blade strength (stiffness achieved by overall blade dimensions or shrouding) rather than dynamic characteristics alone is required to counter the discrete-incident character of these gust loads. Technology advancement? 1) One suggests refinement of ground vortex "snake" mechanism, period, and intensity; and 2) its interaction with cascade unsteady aerodynamics and relevance to gust loads.

B. Upstream Turbulence

In-service distortion generally derives from some upstream separation. Separated flows produce both a time-average (and hence geometric pattern), pressure loss, and a velocity-pressure time unsteady component definable in terms of spectral distributions. Severe random excitation vibration has been observed beyond that due to cascade aerodynamic per se. One infers as the mechanism unsteady incidence excursions increasing cascade separation.

An area for technology advancement is separated flow vibration intensity refinement; discrimination between random excitation directly associated with a turbulent airstream as opposed to a turbulent airstream's triggering of cascade boundary-layer separation by fluctuating incidence.

C. Oscillatory Flows

Sustained oscillatory flows in the inlet, whether or not significant for stall, must be eliminated if their frequency is capable of lower blade mode resonance. Without detailed quantification, it is known for example, that oscillatory flows stemming from incompletely guided turning in inlet ducts and resonant with inlet guide vane first torsional modes are intolerable. Relevant and worthy of further refinement, are definition of frequency and intensity of periodic flows shed by boundaries of distortion screen rings and sectors; and transient characteristics of supersonic flight inlets.

VIII. A Distortion Index for Vibration

The foregoing discussion indicates that a considerable body of data and techniques can be applied to distinguishing distortions capable of generating serious blade vibration from those which do not. They have already contributed to engine development. Their refinement is continuing and growing in scope. But availability does not guarantee application. Distortion definition in terms almost useless for vibration guarantees the technology will be used only too late. Failure of finite foresight to prevent a first-of-its-kind phenomenon is excusable. But it is tragic oversight to describe the sequence of distortion patterns in Fig. 1, for example, by one and the same stall—related index of circumferential distortion, and thereby presumably to permit the half-moon pattern in one installation where no resonance exists to imply success in fatigue avoidance in an installation having the same index with the cruciform pattern. By contrast, the text indicates the vibration significance of the radial distribution including related Fourier series harmonics' magnitude and phasing (sector skewing). By visual inspection one might infer significant data of this type for the crosswind pattern of Fig. 2; but never, without numerical quantification, for the high Mach flight pattern of Fig. 3.

Hence, elementary considerations of blade vibration require that an arbitrary distortion pattern be defined in terms of at least five radial stations and eight harmonics (per revs). A Distortion Excitation Index for Vibration, then, becomes an array of coefficients DEIV = $[Cij]$ where Cij involves a magnitude and a space (phase) φ angular orientation to a common reference.

$$\begin{array}{lcc}
\text{Harmonic} & & 0\quad 1\quad 2\quad 3\quad 4\quad 5\quad 6\quad 7\quad 8 \\
\text{Radial Station} & & \\
\text{DEIV} = & \begin{array}{c}5\\4\\3\\2\\1\end{array} & \begin{bmatrix} C_{0,5} & & \\ & C_{4,4/\varphi 4,4} & \\ & & \\ & & \\ & & C_{8,1}/\varphi_{8,1} \end{bmatrix}
\end{array}$$

where $Cij/\varphi ij$ are based on the definition of $(P_{\text{Ring Avg}}-P\theta)/P_{\text{Face Avg}}$, and $C_{o,j}$ is based on $(P_{\text{Face Avg}}-P_{\text{Ring Avg}})/P_{\text{Face Avg}}$.

The zeroth column of coefficients is at once an alert for blade/vane instability and separated flow vibration; succeeding columns, for blade resonance. The DEIV, together with geometric representations like Fig. 3, provides an alert to discrete sectors capable of producing severe vane separated flow vibration.

This index is merely a formalization of initial steps already mandatory for blade vibration assessment. Its utilization provides, however, an early warning design alert and, more important, precludes interdisciplinary communication breakdown as well as specification blind spots.

Only indirectly could it suggest nonsteady distortion and not at all sledgehammer impact loads in crosswind. The text suggests the former may be of lesser importance; the latter requires a separate, specialized definition beyond what should be discussed here.

IX. Blade Vibration and Steady-State Aerodynamics

Prediction of blade vibration in the presence of inlet distortion may be viewed as a chain multiplication of transfer functions starting with a characteristic scalar $\Delta P/P$; i.e., 1) $(\Delta P/P)$; 2) a geometry-defining distribution function; 3) velocity perturbations at the blade leading edge per unit $\Delta P/P$; 4) nonsteady aerodynamic forces per unit velocity perturbations; and 5) structural stress response per unit excitation force. Transfer function 3 plays a crucial role. It will be noted that the text describes only one example for the previous chain multiplication as a complete, self-contained, first-principles procedure: the distortion-induced stress response of a fan without guide vanes. The more cascades intervening between reference station distortion definition and the blade cascade studied for vibration, the more crucial the role of steady-state aerodynamics in defining the environment for the application of blade vibration characteristics defined in terms of it.

An extensive literature of theoretical analysis and experimental measurement is concerned with attenuation of inlet distortion intensity and pattern as it migrates through, and is itself modified by, successive compressor stages; e.g., Refs 18–20. Investigations of this type address the evolution of distortion patterns stage-by-stage in multistage compressors and specifically as reflected in blade incidence/velocity perturbations; attenuation characteristics as a function of Mach number, stage load-flow characteristics, and stator rotor axial gap. In these topics, concerns for stall and vibration are parallel. Success in prediction refinement of such steady-state aerodynamics topics may well pace the applicability of blade vibration technology.

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